[13] Barnes PAMS 2013

L left Leibniz alg. $\quad d_a : L \to L \quad d_a(x) = ax$

definition of L ~~module~~ module (L a Leibniz algebra)

This is a vector space V and two <u>bilinear</u> maps

$L \times V \ni (x, v) \longrightarrow xv \in V$

$L \times V \ni (x, v) \longrightarrow vx \in V$

satisfying

NOTATION: $\lambda_x : V \to V$, $\quad v \to xv$

$\rho_x : V \to V$, $\quad v \to vx$

- $x(yv) = (xy)v + y(xv) \qquad \lambda_x \lambda_y = \lambda_{xy} + \lambda_y \lambda_x$
- $x(vy) = (xv)y + v(xy) \qquad \lambda_x \rho_y = \rho_y \lambda_x + \rho_{xy}$
- $v(xy) = (vx)y + x(vy) \qquad \rho_{xy} = \rho_y \rho_x + \lambda_x \rho_y$

<u>Example</u>: $L \underset{\text{subalgebra}}{\subset} L'$ (= a Leibniz alg)

$V \underset{\text{abelian ideal}}{\subset} L'$

For more on modules see reference [8] in this paper, which is reference [47] in Rakhimov's survey.
(it has already been posted)

<u>definitions</u> if V is an L-module (L a Leibniz alg)

$\ker V = \{ x \in L : \lambda_x = \rho_x = 0 \}$

$Z(L) = \{ x \in L : \cancel{xy} \; yx = xy = 0 \quad \forall y \in L \}$ ~~this is the "left center"~~

$Ann(L) = \{x \in L : xL = 0\}$ is the left annihilator of L

(Rakhimov calls this the left center — see the survey p. 230)

$Leib(L) = span\{x \in L : x^2 = 0\}$ is the Leibniz kernel of L

(it is an abelian ideal, $Leib(L) \cdot L = 0$, $L/_{Leib(L)}$ is a Lie algebra)

Engel's theorem (for Lie algebras) holds in Leibniz algebras

[1: Th 2] Ayupov & Omirov Proceedings of a Colloq in Tashkent 1998

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[9: Th 7] A Patsourakos Comm. Alg 35 2007 3828–3834

downloaded patsourakos CommAlg 2007 [9].pdf

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Lie algebra theorems (to be generalized to Leibniz algs)

Ado-Iwasawa: For each finite dim'l Lie algebra L over a field F, there is a faithful L-module V.

(meaning that the representation corresponding to the L-module V is one-to-one)

MAIN RESULT is

Theorem 4.4 Let L be a Leibniz algebra of dimension n. Then there exists a faithful L-module V of dimension $\leq n+1$ with 3 properties

(1) The left action λ_x of x on V is nilpotent, if d_x is nilpotent

((2), (3) not stated here)

Remarks on References in the paper

[6] K. Iwasawa Jap. J. Math 1948 Iwasawa1948[6]MR.pdf
AdoThmThesis.pdf

is Iwasawa's proof of Ado's theorem

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[5] G. Hochschild PAMS 1966 Hochschild1966[5].pdf
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[7] Jacobson Book 1962 (Proof of Ado's theorem)
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[2] D. Barnes J. Aust. M. S. 2005 barnesJAusMS2005[2].pdf
MR.pdf

[4] D. Barnes & H.M. Gastineau-Hills MZ 1968
barnesEtAlMZ1968[4].pdf
MR.pdf

are improvements of the Ado-Iwasawa theorem

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[3] D. Barnes arXiv: 1101.3046 2011 Comm Alg
barnesCommAlg2013[3].pdf
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is used in the proof of statement (3) of Theorem 4.4.

It is enough to prove Statements (1) and (2) for purposes of this project.